

AUA Math Test

Sample Questions

November 21, 2012

1. If $2a + 3 = 6$, then $\frac{3}{4a + 6} =$
(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3
2. If $4^{2x+2} = 64$, then $x =$
(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) 2 (E) $\frac{5}{2}$
3. In Figure 1, the area of rectangle $CDEF$ is twice the area of rectangle $ABCF$. If $CD = 2x + 2$, what is the length of AE , in terms of x ?

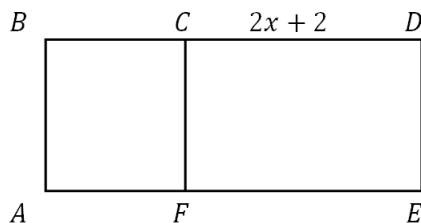


Figure 1

- (A) $2x + 3$ (B) $2x + 4$ (C) $3x + 1$ (D) $3x + 2$ (E) $3x + 3$
4. If $2y^2 + x - 4 = 0$ and $\frac{x}{2} = y^2$, then $x =$
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5
 5. If the first 3 items of a geometric progression are $a - 1$, $a + 3$, and $3a + 1$ for some positive number a , what is the numerical value of the fourth term?
(A) 25 (B) 36 (C) 32 (D) 100 (E) 9

6. In Figure 2, if congruent right triangles ABD and DCA share leg AD , then what is angle x ?

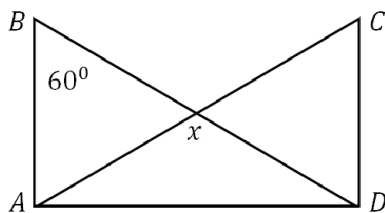


Figure 2

- (A) 90° (B) 100° (C) 110° (D) 120° (E) 130°
7. Which of the following numbers is closest to
- $$\frac{\log_3 81}{\log_4 1024}$$
- (A) 0.6 (B) 0.8 (C) 1.2 (D) 1.3 (E) 0.4
8. If $a \downarrow b = \sqrt[b]{a}$, then $10 \downarrow 3 =$
- (A) 1.12 (B) 1.69 (C) 2.15 (D) 2.71 (E) 3.33
9. Which of the following ordered pairs (x, y) is the solution to the system of equations $x + 2y = 5$ and $x - 2y = 9$?
- (A) $(-7, -1)$ (B) $(-1, 7)$ (C) $(7, -1)$ (D) $(-7, 1)$ (E) $(1, 7)$
10. Which of the following lines is parallel to $y = -2x + 3$ and has a y -intercept of 4?
- (A) $y = -2x + 4$ (B) $y = -2x - 4$ (C) $y = 2x - 4$ (D) $y = 2x + 4$ (E) $y = \frac{1}{2}x + 4$
11. In Figure 3 the area of quadrilateral $ABCD$ is

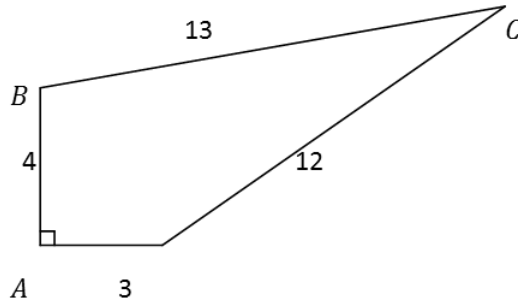


Figure 3

- (A) 32 (B) 33 (C) 34 (D) 35 (E) 36

12. If $f(x) = x^2 + x$ and $g(x) = \sqrt{x}$, then $f(g(3))$ is closest to
 (A) 1.73 (B) 3.46 (C) 4.73 (D) 7.34 (E) 12.00

13. At a certain software company, the cost C of developing and producing a computer software program is related to the number of copies produced, x , and by the equation $C = 30,000 + 2x$. The company's total revenues R is related to the number of copies produced and by the equation $R = 6x - 10,000$. How many copies must the company produce so that the revenue is equal to the cost?
 (A) 5,000 (B) 6,000 (C) 7,500 (D) 9,000 (E) 10,000

14. If the two squares shown in Figure 4 are identical, what is the degree measure of angle exterior CDE ?

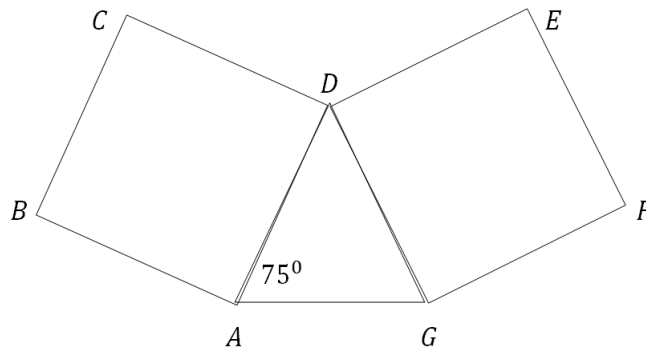


Figure 4

- (A) 120 (B) 135 (C) 150 (D) 165 (E) 175

15. Points $A(\sqrt{2}, 4)$, $B(6, -\sqrt{3})$, and C are collinear. If B is the midpoint of line segment AC , what are the approximate (x, y) coordinates of point C ?

(A) (3.71, 1.13) (B) (3.71, 5.73) (C) (7.41, -7.46) (D) (10.59, -7.46) (E) (10.59, 5.73)

16. Find the values of x satisfying the equation $4 + x^2 = 2x^2 - 5$?

(A) $x = 3$ (B) $x = -3$ (C) $x = \pm 3$ (D) $x = -1$ (E) $x = 1$

17. Which of the following triples can be the lengths of the sides of a triangle?

(A) 2, 3, 5 (B) 1, 4, 2 (C) 7, 4, 4 (D) 5, 6, 12 (E) 9, 20, 8

18. In Figure 5, if the straight line L has a slope of 1 and passes through the origin, which of the following points has (x, y) coordinates such that $\frac{x}{y} > 1$?

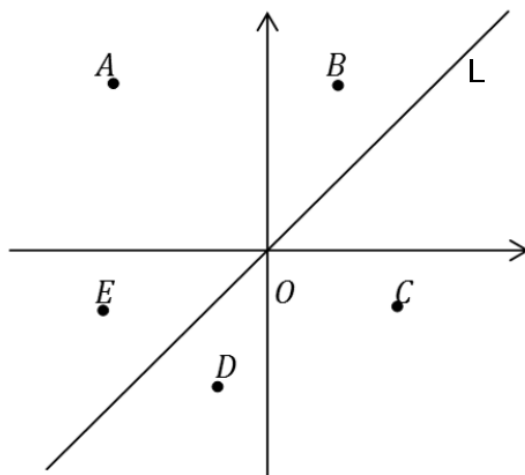


Figure 5

(A) A (B) B (C) C (D) D (E) E

19. On a recent chemistry test, the average (arithmetic mean) score among 5 students was 83, where the lowest and highest possible scores are 0 and 100, respectively. If the teacher decides to increase each student's score by 2 points, and if none of the students originally scored more than 98, which of the following must be true?

I. After the scores are increased, the average score is 85.

II. When the scores are increased, the difference between the highest and lowest scores increases.

III. After the increase, all 5 scores are greater than or equal to 25.

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II and III

20. Calculate the expression $1 - 2\sin^2\theta - 2\cos^2\theta$
 (A) -2 (B) -1 (C) 0 (D) 1 (E) 2
21. If $f(x) = x^2 - 1$, $g(x) = (x - 1)^{(-1)}$, and $x \neq 1$, then $f(x)g(x) =$
 (A) $2x + 1$ (B) $x + 1$ (C) $x - 1$ (D) $x^3 - 1$ (E) $2x - 1$
22. In Figure 6, AE is parallel to BD . What is the length of ED ?

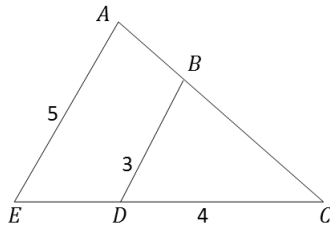
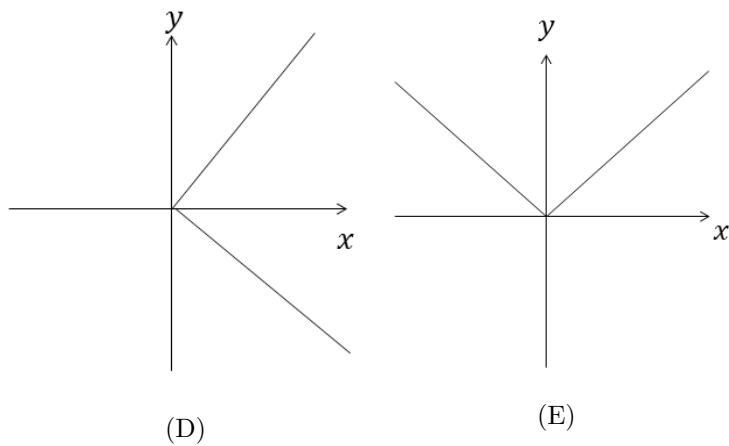
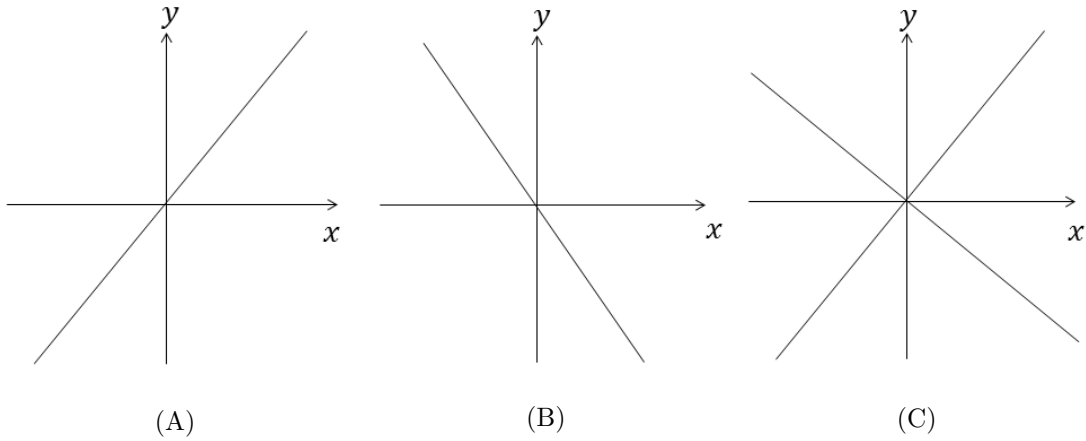


Figure 6

- (A) $\frac{8}{5}$ (B) $\frac{8}{3}$ (C) $\frac{3}{8}$ (D) $\frac{5}{3}$ (E) $\frac{4}{3}$
23. If x and $f(x)$ are real, what is the domain of $f(x) = \sqrt{(x^2 - 4)}$?
 (A) All real numbers
 (B) All x such that $x \geq 2$
 (C) All x such that $x \leq 2$
 (D) All x such that $-2 \leq x \leq 2$
 (E) All x such that $x \leq -2$ or $x \geq 2$
24. What is the area of the triangle with vertices $(1, 1)$, $(3, 1)$, and $(5, 7)$?
 (A) 6 (B) 7 (C) 9 (D) 10 (E) 12
25. If $f(x) = \frac{1}{x}$, and $0 < x < 1$, what is the range of $f(x)$?
 (A) All real numbers
 (B) All real numbers between 0 and 1
 (C) All real numbers greater than 0
 (D) All real numbers greater than 1
 (E) All real numbers greater than or equal to 1
26. The equation $x^2 = y^2$ is represented by which of the following graphs?



27. Calculate $3 - 2 \log_{10} 5 - \log_{10} 4$
 (A) 1.8 (B) 1.5 (C) 2 (D) 1 (E) -1
28. Find the x -intercepts of function $f(x) = 3x^2 - 24x + 45$
 (A) -3, 5 (B) 2, 4 (C) 5, 9 (D) 3, 5 (E) -5, 9
29. Solve for x the following equation $\log_{16} x + \log_4 x + \log_2 x = 7$
 (A) 14 (B) 16 (C) 18 (D) 20 (E) 10
30. Calculate the length of vector \overrightarrow{AB} knowing that its head is $A = (3, -1)$ and its tail is $B = (-2, 4)$.
 (A) 2 (B) $\sqrt{50}$ (C) 1 (D) $\sqrt{3}$ (E) $\sqrt{25}$

31. In the figure below, when \vec{OB} is subtracted from \vec{OA} , what is the length of the resultant vector?

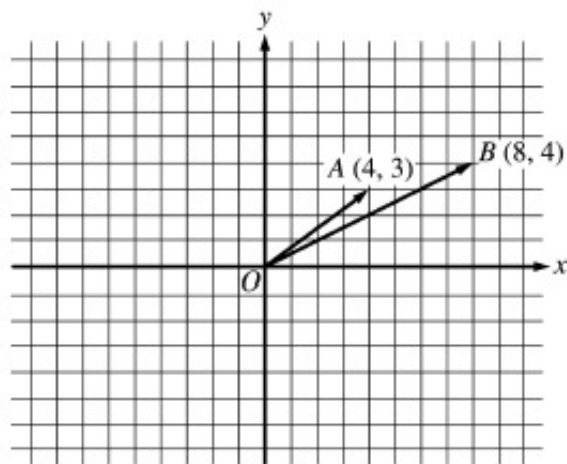


Figure 7

- (A) 3 (B) 4.1 (C) 5 (D) 8.9 (E) 13.9
32. Calculate
- $$\log_b b^{21}(\log_b b^{10} - \log_b 1 - \log_b b)$$
- (A) 210 (B) 221 (C) 198 (D) 212 (E) 189
33. $X = \{2, 4, 6, 8, 10\}$ $Y = \{1, 2, 3, 4, 5, 6\}$ $Z = \{3, 4, 6, 7, 10\}$
 Find $(X \cup Y) \cap Z$
- (A) $\{1, 2, 3, 10\}$ (B) $\{3, 4, 6, 10\}$ (C) $\{7, 10, 11\}$ (D) $\{2, 4, 6, 8\}$ (E) $\{1, 3, 8, 10\}$
34. If $U = \{\text{integers less than } 20\}$ and $P = \{\text{prime numbers less than } 20\}$, then which of the following sets overlaps with P, and is also a part of U?
- (A) $Q = \{\text{even numbers less than } 20\}$
 (B) $R = \{\text{multiples of } 4\}$
 (C) $S = \{\text{factors of } 36\}$
 (D) All of the above
 (E) None of the above
35. The distance from home to school is 280 m. The older brother takes 300 fewer steps than his younger brother to cover the same distance from home to school. The older brother's steps are 30 cm longer than his younger brother's. Determine the older brother's step size.
- (A) 70 cm (B) 60 cm (C) 65 cm (D) 75 cm (E) 80 cm

36. Find the minimal positive solution of the following equation

$$\sin x \cdot \sin(2x) \cdot \sin(3x) = \frac{1}{4} \cdot \sin(4x).$$

- (A) $\frac{\pi}{8}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$ (E) $\frac{\pi}{2}$

37. If $\log \frac{a}{b} + \log \frac{b}{a} = \log(a + b)$, then

- (A) $a + b = 1$ (B) $a \neq b$ (C) $a - b = 1$ (D) $a^2 - b^2 = 1$ (E) $a^2 + b^2 = 1$

38. For the remainder of the division of $x^3 - 2x^2 + 3kx + 18$ by $x - 6$ to be equal to zero, k must be equal to

- (A) 0 (B) 1 (C) 5 (D) -9 (E) -10

39. The side of an equilateral triangle inscribed in a circle is equal to 9 *cm*. Calculate the area of the square inscribed in the same circle (in cm^2).

- (A) 76 (B) 48 (C) 58 (D) 64 (E) 54

40. In Figure 8, each of the 3 circles is tangent to the other 2, and each side of the equilateral triangle is tangent to 2 of the circles. If the length of one side of the triangle is 1, what is the radius of the circles?

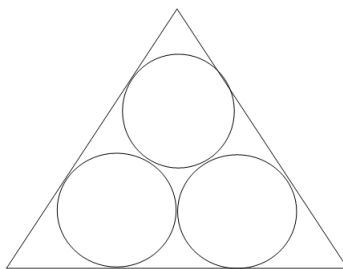


Figure 8

- (A) $\frac{1}{1 + 2\sqrt{3}}$ (B) $\frac{1}{2 + 2\sqrt{3}}$ (C) $\frac{1}{1 + \sqrt{3}}$ (D) $\frac{2}{1 + \sqrt{3}}$ (E) $\frac{2}{1 + 2\sqrt{3}}$